APPLICATIONS
OF POTENTIAL THEORY
IN MECHANICS

Selection of new results

V.I. Fabrikant

To my parents

Моим родителям посвящается

PREFACE

It is not easy to find something new in mathematics. It is difficult to find something new in something very old, like the Potential Theory, which was studied by the greatest scientists in the past centuries. And it is extremely difficult to make this find on an elementary level, with no mathematical apparatus involved which would be considered new even in the times of, say, Poisson. This is exactly what the author claims to have done in this book: a new and elementary method is described for solving mixed boundary value problems, and their applications in engineering. The method can solve *non-axisymmetric problems* as easily as axisymmetric ones, *exactly and in closed form*. It enables us to treat *analytically* non-classical domains. The major achievements of the method comprise the derivation of explicit and elementary expressions for the Green's functions, related to a penny-shaped crack and a circular punch, development of the Saint-Venant type *theory of contact and crack problems for general domains*, and investigation of various *interactions* between cracks, punches, and external loadings. The method also provides, as a bonus, a tool for exact evaluation of various two-dimensional integrals involving distances between two or more points. It is believed that majority of these results are beyond the reach of existing methods.

For over twenty years, since being a graduate student in Moscow, the author had been perplexed by an inconsistency between various solutions to the problems in Potential Theory and the way those solutions have been obtained, namely, that the solution was quite elementary, while the apparatus used was very complicated, involving various integral transforms or special functions expansions, which are beyond the comprehension of an ordinary engineer. The author's search for a new method was based on the conviction that an elementary result should be obtainable by elementary means. The method has been found and is presented here in detail. One may just wonder why the method was not discovered at least a century ago.

The book is addressed to a wide audience ranging from engineers, involved in elastic stress analysis, to mathematical physicists and pure mathematicians. While an engineer can find in the book some elementary, ready to use formulae for solving various practical problems, a mathematical physicist might become interested in new applications of the mathematical apparatus presented, and a pure mathematician might interpret some of the results in terms of fractional calculus, investigate the group properties of the operators used, or, having noticed the fact that no attention is paid in the book to a rigorous foundation of the method, might wish to remedy the situation. Due to the mathematical analogy between mixed boundary value problems in elasticity and in other branches of engineering science, *the book should be of interest to specialists in electromagnetics, acoustics, diffusion, fluid mechanics*, etc. Though several such applications have been published by the author, the space considerations did not allow us to include them, but references are given at appropriate places in the book.

The book is *accessible to anyone* with a background in university undergraduate calculus, but should be of interest to established scientists as well. Though the method is elementary, the transformations involved are sometimes very non-trivial and cumbersome, while the final result is usually very simple. The reader who is interested only in application of the general results to his/hers particular problems may skip the long derivations and use the final formulae which require little effort. The reader who wants to master the method in order to solve new problems has to repeat the derivations which are given in sufficient detail. The exercises are important in this regard. They vary from very simple to quite difficult. Some can be used as a subject for a graduate degree thesis.

The book is based entirely on the author's results, and this is why the work of other scientists is mentioned only when such a quotation is inevitable for some reason, like numerical data needed to verify the accuracy of approximate results, comparison with existing results, or pointing out some errors in publications. There are several books and review articles presenting an adequate account of the state-of-the-art in the field. Appropriate references are given for the reader's convenience. The purpose of this book was neither to repeat nor to compete with them.

The development of the method can by no means be considered completed, this is just a beginning. The results presented in the book may be compared to the tip of an iceberg, taking into consideration numerous applications which are still to come. The solution of fundamental problems in a simple form enables us to consider various more complicated problems which were not even attempted before. The *method can be expanded* to spherical, toroidal and other systems of coordinates, so that more complex geometries may benefit from it. The method proved useful in the generalized potential theory as well. Some of these results, though already published by the author, could not be included in the book due to severe restrictions on the book volume.

For the reader's convenience, it was attempted to make each chapter (and section, wherever possible) self-contained. The reader can skip several sections and continue reading, without loosing the ability to understand material. On the other hand, this resulted in repetition of some definitions and descriptions. The author thinks that the additional convenience is worth several extra pages in the book.

The author is grateful to Professor J.R. Barber from Michigan, Professor B. Noble from England, and Professor J.R. Rice from Harvard who agreed to read the manuscript and expressed their opinion.

The book contains so much new material that some misprints and errors are inevitable, though every effort was made to eliminate them. The author would be grateful for every communication in this regard. All the readers' comments are welcome. The address is

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