CONTACT AND CRACK PROBLEMS IN LINEAR THEORY OF ELASTICITY

By **V.I. Fabrikant**

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INTRODUCTION

This book may be considered as a logical continuation of the two previously published books (V.I. Fabrikant, *Applications of Potential Theory in Mechanics*, Kluwer Academic, 1989) and (V.I. Fabrikant, *Mixed Boundary Value Problems of Potential Theory and Their Applications in Engineering*, Kluwer Academic, 1991), where a new and elementary method was described for solving mixed boundary value problems. The method can solve *non-axisymmetric problems* as easily as axisymmetric ones, *exactly and in closed form*. It enables us to treat *analytically* non-classical domains. The method also provides, as a bonus, a tool for exact evaluation of various two-dimensional integrals involving distances between two or more points.

The main emphasis of the first book was on solid mechanics problems. In the second book we described various applications of the new method to electromagnetics, acoustics and diffusion. Also included in the second book were some results in fracture mechanics and elastic contact problems, which were obtained later and could not be included in the first book.

There are numerous books on contact problems (Galin, 1951, Rvachev and Protsenko, 1977; Mossakovskii *et al.*, 1985), there are also books devoted to the crack problems (Cherepanov, 1974; Kassir and Sih, 1975). There seems to be no titles, giving exhaustive treatment of both contact and crack problems in one book. Why this is so, is beyond my comprehension. Indeed, from mathematical point of view, there is not much difference between them: a contact problem is characterized by a displacement, prescribed inside certain domain, with the stress being zero outside, while a crack problem is characterized by a stress prescribed inside a domain, with the displacement vanishing outside. In a way, one problem may be considered as inverse of the other.

This seems to be the first book, giving the state-of-the-art description of both contact and crack problems. The same mathematical apparatus, developed by the author, is used to solve both. Majority of presented solutions is exact and expressed in terms of elementary functions. One may argue, that these elementary solutions are not needed in the age of powerful computers. This is just not so: the most powerful computers are still quite bad in the cases of poor convergence and can not handle singularities directly. In addition, elementary solutions serve as excellent benchmark examples to verify the quality of new numerical methods or a new software.

The book is addressed to a wide audience ranging from engineers to mathematical physicists. While an engineer can find in the book some elementary, ready to use formulae for solving various practical problems, a mathematical physicist might become interested in new applications of the mathematical apparatus presented. The book should be of interest to specialists in electromagnetics, acoustics, diffusion, solid and fluid mechanics, etc.

The book is *accessible to anyone* with a background in university undergraduate calculus, but should be of interest to established scientists as well. Though the method is elementary, the transformations involved are sometimes very non-trivial and cumbersome, while the final result is usually very simple. The reader who is interested only in application of the general results to his/her particular problems may skip the long derivations and use the final formulae which requires little effort. The reader, who wants to master the method in order to solve new problems, has to repeat the derivations which are given in sufficient detail.

The book is based entirely on the author's results, and this is why the work of other scientists is mentioned only when such a quotation is inevitable for some reason, like numerical data needed to verify the accuracy of approximate results, comparison with existing results, or pointing out some errors in publications. There are several books and review articles presenting an adequate account of the state-of-the-art in the field. Appropriate references are given for the reader's convenience. The purpose of this book was neither to repeat nor to compete with them.

For the reader's convenience, it was attempted to make each chapter (and section, wherever possible) self-contained. The reader can skip several sections and continue reading, without loosing the ability to understand material. On the other hand, this resulted in repetition of some definitions and descriptions. The author thinks that the additional convenience is worth several extra pages in the book.

The book contains *global* variables, which denote the same quantity throughout the book, for example, $l_1(\cdot,\cdot,\cdot)$ $l_2(\cdot,\cdot,\cdot)$, Λ, etc. There is only limiting amount of characters in the Latin and Greek alphabets, while the number of parameters and notations used in the book is much greater, so inevitably some characters are used as *local* variables to denote several different quantities. For example, δ denotes in some sections the angle of inclination, while in other sections it denotes $\left(\rho\rho_0 e^{i(\phi-\phi_0)}\right)^{1/2}$. Hopefully, no character is used to denote 2 different quantities in the same section.

The book consists of 6 chapters. The first chapter describes the mathematical foundations of the method, with some applications. Two chapters are devoted to crack problems and 2 chapters describe the contact problems. Each subject is divided in two parts: the fundamental problems and the advanced ones. The most important results from the previous 2 books are repeated in a concise form, while the new results of the past 12 years are given in detail.

Several Appendices in the book contain various mathematical formulae (derivatives, integrals, etc.), which are not available in other mathematical reference books. Chapter 6 is completely devoted to such results, giving various derivatives, one- and two-dimensional integrals. The method developed in the book can be generalized for solving mixed boundary value problems for piezo-electric or piezo-magnito-electric bodies. Numerous publications in this field are available in the literature.

The book contains so much new material that some misprints and errors are inevitable, though every effort was made to eliminate them. The author would be grateful for every communication in this regard. All the readers' comments are welcome. The author's present mailing address is:

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